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"LINEAR THEORY OF THE POLARIZED RELAY"

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1. Introduction

1935 saw the rough construction of a general linear theory of two-position polarized relays, the means employed being the graphical method (vide G. A. Ostroumov's article " A Method of Calculating a Polarizing Relay", in Nauchno-teckhnicheskiy Sbornik, Leningrad. Elektrotekhn. Instituta Svyazi (Scientific Technical Symposium of the Leningrad Electrotechnical Institute of Communication). No. 9, pp 58-64, 1935). In the course of the following years this primitive theory withstood experimental tests (vide A. Zhelnova's "The Frequency Characteristic of a Relay for a Sinusoidal Signal". A Student's Diploma Work at the Saratovsk State U, 1937/38). The tests established the general trustworthiness of the theory, but revealed systematic deviations.

Therefore the necessity arose of building, on principally the same foundation, a new and more rigid theory. This theoretical foundation can be formulated thus: the structure and analysis of the frequency characteristic of the relay, disregarding nonlinear distortions. The obtaining of constructive conclusions concerning the ways and limits of possible improvement of this type of relay is, therefore, the purpose of this analysis.

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2. Scheme of Construction

For definiteness we shall represent the construction of the relay or similar apparatus (for example, undulator) by some sort of scheme as in Figure 1 (all figures are annexed). In it, A is a yoke of laminated transformer steel; B is the ferromagnetic laminated part of the armature; C is its axis of rotation; D_1 and D_2 are stops; G ~~is~~ are the gaps ~~clearance between the iron~~; F is a permanent magnet or electromagnet supplied with direct current.

3. Simplifying Assumptions

A. Linearity of the system: 1) Movements of the armature between the stops D_1, D_2 ~~is~~ are to be considered "small" in comparison with the length of the air gap G. 2) The current strength of the signal in the region of interest to us in this analysis can be considered "small" within the limits defined by the law in Equation (1) below. Then, in accordance with Hopkins' formula, let us assume as follows:

a) magnetic flux Φ in the active gaps G equals:

$$\left.\begin{aligned}
&\Phi=(1-\sigma)\frac{0.4\pi\omega}{R_m-x/S}(I_0+I)=\Phi_0+\frac{\partial\Phi}{\partial x}+\frac{\partial\Phi}{\partial I}I\\
&\Phi_\theta=(1-\sigma)\frac{0.4\pi\omega}{R_m}\ ;\ \frac{\partial\Phi}{\partial x}=\frac{(1-\sigma)0.4\pi\omega I_0}{SR_m^2}=\frac{\Phi_0}{SR_m}\\
&\frac{\partial\Phi}{\partial I}=\frac{(1-\sigma)0.4\pi\omega}{R_m}=\frac{1-\sigma}{\omega}\cdot L\cdot 10^8
\end{aligned}\right\}\quad(1)$$

b) ~~for~~ force F of attraction of the armature for the side of the pole shoe equals:

$$F=\frac{\Phi^2}{8\pi S}=F_0\pm\frac{\partial F}{\partial x}x\pm\frac{\partial F}{\partial I}I+F_0\pm\frac{C}{2}x\pm\frac{N'}{2}I$$

$$F_0=\frac{\Phi_0^2}{8\pi S};\quad 2\frac{\partial F}{\partial x}=C=2\frac{\partial F}{\partial\Phi}\cdot\frac{\partial\Phi}{\partial x}=\frac{\Phi_0}{2\pi S}\cdot\frac{\Phi_0}{R_mS}=\frac{4F_0}{R_mS}.\quad(2)$$

$$2\frac{\partial F}{\partial I}=N'=2\frac{\partial F}{\partial\Phi}\cdot\frac{\partial\Phi}{\partial I}=2\frac{\Phi_0}{R_mS}(1-\sigma)\omega\cdot\frac{1}{10}$$

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In these formulas sigma σ signifies the coefficient of scattering (diffusion); R_m is the magnetic resistance of the magnetic conductor of alternating-current signals when the armature is in the neutral position; I_o is the fictional direct current in the signal winding which would cause the same flux as that due to the permanent magnet; X is the displacement of the armature from neutral, or variation in the gaps G; S is the area of each pole shoe; L is the coefficient of ~~self~~ inductance of the signal winding for neutral position of the armature. All quantities are in practical units: cm, gram, sec, volt, amp, ohm.

In electro-acoustics, the parameter C is called negative resilience of the magnetic field; N^1 is the first (direct) coefficient of ~~the~~ electro-mechanical ~~bond~~ coupling.

B. Other assumptions: 1) Let us consider the core of the electro-magnet and the armature to be laminated ('burdened') so that we may neglect the influence of Foucault currents. 2) The collisions of the armature against the stops D will be considered absolutely ~~non-electric~~ inelastic. 3) The moving armature system will be considered as a mechanical constant system centered (~~but~~ and not distributed) and "reduced" to one point. Let us choose the center of gravity of the area S as the point of reduction. 4) Keeping in mind ~~the~~ problems of rapid action, let us neglect friction in the bearings in comparison with inertia and negative elasticity of the magnetic field. 5) Let us assume that the signal has the form of telegraphic dots smoothed to sinusoidal form with the preceding elements of the communication channel. (Note: The sinusoidal form of the dot signal is selected as the most simple form, which facilitates greatly otherwise difficult mathematical computations. Extremely cumbersome calculations give almost the same general result for dots of box-like form). 6) the generator (oscillator) has purely active constant internal resistance R (amplifier tube in the rectilinear region of ~~the~~ its characteristics) will be taken as the source ~~of~~ signals. 7) The resistance (ohmic) of the signal winding is assumed to be small in comparison with the signal generator's ~~oscillator's~~ resistance, or the resistance of the winding is considered already included in the resistance R.

4. Basic Equations

A. The armature in motion: When the armature rotates around axis C, the unbalanced part of the force F in Equation (2) acts upon the armature according to the following formaula:

$$m\ddot{x} = Cx + N'I. \tag{3}$$

Here m signifies the reduced mass of the armature and the two dots over x mean the second derivative with respect to time.

The electrical system of the relay can be schematically represented as shown in Figure 2. Acting upon it are: a) electromotive force of the signal E.exp (jwt) and b) emf due to the variation of magnetic flux during armature motion:

$$-\omega\frac{\partial\Phi}{\partial x}\cdot 10^{-8}\cdot x = -N''x.$$

$$\text{Here } N^{1} = (1-\sigma)\,\omega\frac{\partial\Phi}{\partial x}\cdot 10^{-8} = \frac{1-\sigma}{R_m S}\cdot\omega\,\Phi_0\cdot 10^{-8} = kN' \tag{4}$$

$$k = 10^{-7}\,\frac{volt\cdot sec}{erg}$$

In electroacoustics, the parameter N^{11} is called the inverse coefficient of ~~the~~ electromechanical coupling ~~bond~~.

Besides these two emf's it is necessary to keep in mind the voltage drop:

$$RI + L\dot{I} = E.\exp(jwt) - N^{11}x \tag{5}$$

(Note: When utilizing the symbolic description together with the algebraic, we take the real part of the complex number as the actual observable value of the periodic quantity).

From (3) and (5) we obtain the basic differential equations describing the movement of the armature:

$$\dddot{x}+\frac{R}{L}\ddot{x}+\left(\frac{N'N''}{mL}-\frac{C}{m}\right)\dot{x}-\frac{RC}{Lm}x=\frac{EN'}{mL}\cdot e^{j\omega t} \qquad (6)$$

If we set here:

$$N^{I}N^{II}=(N^{I})^2k=N^2 \quad \text{and} \quad 0\leqslant\frac{N^2}{LC}=(1-\sigma)^2\leqslant 1$$

then we obtain:

$$\dddot{x}+\frac{R}{L}\ddot{x}-\frac{C}{m}\left(1-\frac{N^2}{LC}\right)\dot{x}-\frac{CR}{mL}x=\frac{EN'}{mL}\cdot e^{j\omega t} \qquad (7)$$

For the assumptions adopted, this is an ordinary third-order linear differential equation with constant coefficients relative to the unknown **x**, the displacement of the armature from the neutral position.

B. The armature at rest on the stop: When the armature presses the stop, Equation (5) is simplified:

$$RI+L\dot{I}=E.\exp(j\omega t) \qquad (8)$$

In Equation (3) the left side is ~~interchanged during this~~ replaced by the reaction of the stop.

C. The characteristic equations: 1) Basic transformation: Setting $x=A.\exp(pt)$ and discarding for the present the right side of Equation (7) we find for the free ($E=0$) motion of the armature the following equation

$$\left.\begin{aligned}&A.\exp(pt)\cdot\left[p^3+\frac{R}{L}p^2-\frac{C}{m}\left(1-\frac{N^2}{CL}\right)p-\frac{RC}{Lm}\right]=0\\ \text{or}\quad &p^3+\frac{R}{L}p^2-\frac{C}{m}\left(1-\frac{N^2}{CL}\right)p-\frac{RC}{Lm}=f(p)=0\end{aligned}\right\}(9)$$

This equation is the characteristic for ~~the differential equation~~ equation (7) and can be expressed thus:

$$f(p)=(p-p_1)(p-p_2)(p-p_3)=p^3-(p_1+p_2+p_3)p^2+(p_2p_3+p_3p_1+p_1p_2)p-p_1p_2p_3=0 \qquad (10)$$

Setting $I=B.\exp(qt)$ and discarding for the present the right side of Equation (8), we obtain for the armature pressed against the stop (within the limits of the assumptions adopted above) the following characteristic for ~~Equation~~ (8):

$$B.\exp(qt)\cdot(R+Lq)=0\ ;\quad q=-\frac{R}{L}\ . \tag{11}$$

2) Expressing the roots by constructive parameters; limits of the numerical values of the roots and of the coefficients of the characteristic equations: Comparing Equations (9) and (10) we find the following relations

$$-\infty < p_1+p_2+p_3=\Sigma=-\frac{R}{L}<0\ ;\ +\infty > p_1p_2p_3=\Pi=\frac{RC}{Lm}>0$$

$$+\infty > \frac{C}{m}=-\frac{\Pi}{\Sigma}=\frac{p_1p_2p_3}{-(p_1+p_2+p_3)}>0\ ;\ -\infty < p_2p_3+p_3p_1+p_1p_2=$$

$$(p_1p_2p_3)\cdot\left(\frac{1}{p_1}+\frac{1}{p_2}+\frac{1}{p_3}\right)=\Pi H=\frac{C}{m}\left(1-\frac{N^2}{LC}\right)\leqslant 0\ . \tag{12}$$

$$1\geqslant(p_1+p_2+p_3)\cdot\left(\frac{1}{p_1}+\frac{1}{p_2}+\frac{1}{p_3}\right)=1-\frac{N^2}{LC}=1-(1-\sigma)^2\geqslant 0.\quad 1\geqslant\frac{N^2}{LC}=(1-\sigma)^2\geqslant 0\ ;\ H=\frac{1}{p_1}+\frac{1}{p_2}+\frac{1}{p_3}\ .$$

From consideration of these relations it follows that one of the three roots, say p_1, must be positive and the remaining two must be negative or complex conjugates with negative real part.

For the first of these two possibilities we see $0 > p_2 > p_3 > -\infty$;

$$-p_2/p_1=\alpha\ ;\ -p_3/p_1=\beta\ ;\ +\infty>\beta>\alpha>0\ .$$

then on the α,β-plane it is possible to set the limits of possible variations of the roots in accordance with the inequalities ~~Equations~~ (12). These limits are indicated in Figure 3.

Setting the complex roots $-p_2/p_1 = a + jb$ and $-p_3/p_1 = a - jb$, we obtain, in similar fashion, on the a, b-plane the limits shown in Figure 4.

From Equation (11) it follows that:

$$-\infty < q = -\frac{R}{L} < 0\ . \tag{13}$$

From these formulas it is obvious that the roots of the characteristic equatons are expressed ~~expressing~~ uniquely by constructive parameters of the apparatus.

The parameters of the apparatus are expressed by the roots only as paired ratios (R/L, C/M) or triple ratios (N^2/LC).

D. The equations of motion: the actual motion of the armature is the sum of three free motions and a forced motion, according to the following equation:

$$x = A_1 e^{p_1 t} + A_2 e^{p_2 t} + A_3 e^{p_3 t} + D e^{j\omega t} \qquad (14)$$

Here, by virtue of (7) and (9), we have:

$$D e^{j\omega t} \cdot f(j\omega) = \frac{N'E}{mL} \cdot e^{j\omega t} \qquad (15)$$

or $Df(j\omega) = \frac{N'}{mL} \cdot E$.

E. The equation of the extra-current: While the armature is immobile at the stop, the current in its winding is described by Formulas (8) and (11), which are transformed into the following equation:

$$I = \frac{E}{R + j\omega L} \cdot e^{j\omega t} + I_0 e^{-Rt/L} \qquad (16)$$

5. The General Case

A. Initial conditions: 1) Determination of the steady state periodic regime: Equations (14) and (16) describe the behavior of the apparatus when the armature is moving and when it is at rest, respectively. They are related to each other through Equation (3) at the moment of transition from one state (motion) to another (rest), which gives us a 'boundary' condition.

Let us analyze the case of ~~the~~ steady state conditions ~~regime~~ of operation ~~work~~, where the motion of the armature is strictly a periodic function of time with the fundamental angular ~~circular~~ frequency w of the signal.

2) At the beginning of armature movement: The motion of the armature is begun at the moment of time t = 0 when the armature is still at the stop and its displacement x equals y, when its velocity $\dot{x}$ is zero and when its

acceleration $\ddot{x}$ (or, ~~differently~~, the reaction of the stop) passes through the zero value. Keeping in mind Equation (14) we can write:

t = 0

$$\left.\begin{aligned} x &= A_1 + A_2 + A_3 + D = y \\ \dot{x} &= A_1 p_1 + A_2 p_2 + A_3 p_3 + Dj\omega = 0 \\ \ddot{x} &= A_1 p_1^2 + A_2 p_2^2 + A_3 p_3^2 - D\omega^2 = 0 \end{aligned}\right\} \quad (17)$$

For the moment $t = \pi/\omega$, exactly at semiperiod, we must write, according to Equations (3) and (16), as follows:

$$\left.\begin{aligned} m\ddot{x} &= C(-y) + \frac{N'E}{R+j\omega L} e^{j\omega t} + N'I_0 e^{-Rt/L} = \\ &= -Cy - \frac{N'E}{R+j\omega L} + N'I_0 e^{-R\pi/L\omega} = 0 \end{aligned}\right\} \quad (18)$$

3) Collision against the stop: the armature, moving according to Equation (14), at a certain moment of time $t = t_1$ reaches the opposite stop and its displacement becomes $x = -y$. At this moment of time a current I flows in the winding of the apparatus; this I is the one described in Equation (3). After a ~~n extremely~~ completely ~~non~~-inelastic collision the armature stops and Equation (14) no longer holds. The ~~c~~urrent I satisfies, as before, Equation (16). Therefore it is necessary to write for this moment the following:

$t = t_1 = \varphi/\omega$:

$$x = A_1 e^{p_1\varphi/\omega} + A_2 e^{p_2\varphi/\omega} + A_3 e^{p_3\varphi/\omega} + De^{j\varphi} = -y;$$

$$m\ddot{x} = m\left(A_1 p_1^2 e^{p_1\varphi/\omega} + A p_2^2 e^{p_2\varphi/\omega} + A_3 p_3^2 e^{p_3\varphi/\omega} - D\omega^2 e^{j\varphi}\right)$$

$$= -Cy + \frac{N'E}{R+j\omega L} e^{j\varphi} + N'I_0 e^{-R\varphi/L\omega}.$$

Here $0 \leq \omega t_1 = \varphi \leq \pi$. The smaller φ, the faster the relay operates. In practice the quantity $1 - \varphi/\pi$ is called the "coefficient of stability" of the apparatus's operation.

4. Review of the results obtained: Equations (15) and (17) determine whether the armature breaks off from the stop and whether it will generally be moved under the action of the signal E. The first Equation in (19) determines whether the armature reaches the opposite stop and if reached, whether in the course of the first semiperiod after the beginning of the motion (that is, whether actually φ is less than π). If the armature reaches the stop in the course of the first semiperiod, then the second Equation in (19) determines the force of the extra-current I_o arising in this case. Finally, Equation (18) tells actually whether this extra-current permits a new breaking away of the armature from the stop exactly a full period after the first start of the armature's movement.

B. Formulating the problem on the construction of the frequency characteristic: Formulas (15), (17), (18), (19) describe the behavior of the apparatus for a steady periodic ~~regime~~ of operation. They consist of a system of 7 simultaneous equations. They are sufficient for finding the family of 'exploitational' parameters A_1, A_2, A_3, D, I_o, φ , E in terms of the constructive parameters Y, R, L, m, C, N^2 (or σ) for a frequency w of the signal.

Eliminating from these 7 equations all unknowns, except E, one can find the dependence of this quantity upon the frequency of the signal for any given values of the constructive parameters; that is, one can construct the frequency characteristic of the apparatus.

Some of these equations have the imaginary ~~multiple~~ coefficient j which defines the complex character of the quantities D and E (and also A_2 and A_3 if the roots p_2 and p_3 are complex conjugates). Therefore the strength of the signal E is determined as to both magnitude and phase.

Some of these equations contain the unknown φ in the power exponent. Therefore, these equations are transcendental, that is, they can possess an infinite number of solutions.

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C. Consideration of ~~the~~ nonperiodic conditions ~~regime~~: We see that for the assumed steady stable periodic condition ~~regime~~ one can observe for a given frequency not any, but just certain definite values of the angle φ corresponding to definite distinct strengths ~~of the~~ signal E. For the other strengths, ~~of the~~ signal the conditions ~~regime~~ cannot be periodic since Equation (18) will be violated. In each succeeding semiperiod the armature's movement will occur with increasing lag or increasing lead as compared with the preceding semiperiod (but, perhaps, alternately lag and lead).

Thus we can expect a family, perhaps consisting of an infinite number, of frequency characteristics corresponding to the assumed exactly periodic steady state condition ~~regimes~~. To each point of each characteristic there will correspond its value of the angle φ.

6. The Partial Case. The Critical Characteristic.

Of all the frequency characteristics of the apparatus one will possess the most important value. This is the "critical characteristic" corresponding to the critical, or limiting value of the angle $\varphi = \pi$. The armature, hardly having ~~scarcely~~ collid~~ing~~ed in~~non~~-elastically against the stop quickly moves in reverse.

(Note: In spite ~~of~~ the instantaneousness of the contact of the armature with the stop, such a contact (may be) ~~is~~ sufficient to discharge the condenser. This discharge can cause ~~ensure~~ the operation of the following apparatus in the communication channel). The strength ~~of the~~ signal that provides ~~ensures~~ such ~~an~~ operation of the apparatus is a minimum. Any ~~Each~~ stronger signal ensures better transmission than the signal corresponding to the critical characteristic. Any ~~Each~~ weaker signal breaks off the normal operation.

To set up the equation of the critical characteristic one must use Formula (15) and the first two formulas in (17). The third Formula in (17) is unsuitable, since the armature can be ~~move with~~ accelerat~~ion~~ed even in the initial moment after collision. Taking into consideration the signs of

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accelerations, in place of this formula (19) we find:

$$\left.\begin{aligned} &(\ddot{x})_{t=\pi/\omega} + (\ddot{x})_{t=0} = 0 \ , \\ &A_1 p_1^2 (e^{p_1\pi/\omega}+1) + A_2 p_2^2 (e^{p_2\pi/\omega}+1) + A_3 p_3^2 (e^{p_3\pi/\omega}+1) = 0 \ . \end{aligned}\right\} \qquad (20)$$

Instead of the first equation from the group in (19) we now find:

$$A_1 e^{p_1\pi/\omega} + A_2 e^{p_2\pi/\omega} + A_3 e^{p_3\pi/\omega} - D = -y \ . \qquad (21)$$

Thus we obtain a system of 5 simultaneous equations with 5 unknowns: A_1, A_2, A_3, D, E. The previous unknown φ now has the definite value π; the unknown strength of the extra-current I_0 drops away because now the armature does not press against the stop in accordance with Equation (16).

Solving this system for E_1 we obtain the equation for the critical frequency characteristic corresponding to a steady periodic ~~regime~~ condition:

$$E = \frac{y m L}{N'} \cdot f(j\omega) \cdot \frac{1}{Q}$$

$$Q = 1 - j\omega \cdot \frac{\begin{vmatrix} p_1^2(e^{p_1\pi/\omega}+1) & p_2^2(e^{p_2\pi/\omega}+1) & p_3^2(e^{p_3\pi/\omega}+1) \\ e^{p_1\pi/\omega}+1 & e^{p_2\pi/\omega}+1 & e^{p_3\pi/\omega}+1 \\ p_1 & p_2 & p_3 \end{vmatrix}}{\begin{vmatrix} p_1^2(e^{p_1\pi/\omega}+1) & p_2^2(e^{p_2\pi/\omega}+1) & p_3^2(e^{p_3\pi/\omega}+1) \\ e^{p_1\pi/\omega}+1 & e^{p_2\pi/\omega}+1 & e^{p_3\pi/\omega}+1 \end{vmatrix}} \quad [sic!]$$

$$= 1 + j\frac{\omega}{T} \qquad (22)$$

The first coefficient represents a constant independent of frequency; the first and second represent the main variable part of the frequency characteristics; the third, containing ~~an operator~~ determinants, ~~represents~~ is in the range of frequencies that interest us nothing more than a correction factor.

7. Scheme of Comparison

A. Ideal characteristic: The construction of the frequency characteristic does not determine whether a good or bad apparatus is analyzed and how

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to improve it. It is necessary to have a standard with which one could compare for each frequency the operation of a given apparatus.

As such a standard of comparison we select an arrangement that ~~keeps~~ retains only the most essential elements: E, R, m, y. The remaining elements of the scheme we assume equal to zero:

$$L \rightarrow 0; \; C \rightarrow 0; \; \sigma \rightarrow 0; \qquad (22^1)$$

As for the parameter N^2, let it acquire its ~~optimum~~ extremum value for each frequency of the signal; it is determined thus:

$$\frac{\partial(|E|^2/R)}{\partial(N^2)} = 0 \, . \qquad (23)$$

The ~~capacity of~~ power required the signal ~~oscillator~~ generator here ~~must be a~~ minimum. It is possible to show that with these assumptions the ideal frequency characteristic is expressed by the following equations:

$$\frac{E_{oi}^2}{2R} = P_{min} = my^2\omega^3 k \, ; \quad E_{oi} = y\sqrt{2Rkm\omega^3} \, . \qquad (24)$$

Here E_{oi} signifies the absolute value of the amplitude of emf signal strength in the ideal circuit ~~scheme of comparison~~. Or:

$$P_{min} = \frac{1}{2} m (\omega y)^2 \cdot 2 \cdot \omega \cdot k \, .$$

Here the first coefficient represents the maximum kinetic energy of the mass m. The third coefficient determines the speed of variation of the kinetic energy.

B. The reduced characteristic: the frequency characteristic in (22) $E = E(w)$ can be brought closer to the ideal frequency characteristic in (24) or be taken farther from it, depending upon the frequency and constructive parameters of the apparatus.

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In order to possess a conscious control of these parameters, let us introduce the concept of "reduced" frequency characteristic by means of the dimensionless factor ξ, defined by the following ratio:

$$\xi = E/E_{oi} \ . \tag{25}$$

Then the reduced frequency characteristic is expressible thus:

$$\xi = \xi(\omega \cdot p_1 \cdot p_2 \cdot p_3) \ . \tag{26}$$

In particular, the main part of the critical characteristic (22) is rewritten by taking expression (12) into account and takes the following form

$$\left.\begin{aligned} \xi' &= \frac{mL \cdot f(j\omega)}{N'\sqrt{2Rkm\omega^3}} = \sqrt{\frac{1}{2}\frac{m}{C}\,\frac{LC}{kN'^2}\cdot\frac{L}{R}}\cdot\frac{f(j\omega)}{\omega^{3/2}} \\ &= \frac{1}{(1-\sigma)\sqrt{2p_1p_2p_3}}\cdot\frac{f(j\omega)}{\omega^{3/2}} \end{aligned}\right\} \tag{27}$$

Setting here $w \to 0$ and $w \to \infty$, we can show that the reduced frequency characteristic both for the lowest frequency (0) and also for the highest (infinity) frequency of the signal diverges to infinity according to the $\pm$ 3/2 power law ($\infty^{\pm 3/2}$). For some middle frequency of the signal the reduced sensitivity of the apparatus is an extremum (a maximum).

8. Determination of the Maximum Reduced Sensitivity.

Computing, according to Equation (27), the square of the parameter and equating to zero the derivative of this square with respect to frequency, we find ~~a~~ that value of frequency, W_o ~~that~~ which corresponds to maximum reduced sensitivity. This value W_o will be called "exploitation frequency". Substituting the value of the exploitation frequency into expression (27), we determine the value of the maximum reduced sensitivity as a function of α, β, a, b or p_1, p_2, p_3, or, finally, of the constructive parameters of the apparatus according to Formula (12).

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9. Dimensionless Parameters of the Apparatus

In selecting these parameters it is practical and convenient that we keep the same group of independent dimensional ratios:

$$\sigma, \; R/L\omega_0, \; C/m\omega_0^2 \; . \qquad (27^1)$$

In tables 1, 2, 3 and in Figure 5 are several calculated values of the dimensionless parameters. Here we have introduced the designation:

$$(W_0/p_1)^2 = \approx \qquad (27')$$

Figure 6 gives the value of the reduced sensitivity as a function of the lumped parameter CL^2/mR^2. (Note: This part of the work was completed by Kotel'nikov and Laschinskiy, students on the mathematical physics faculty of the Moscow State University).

10. Practical Results

From a consideration of tables 1, 2, 3 and Figure 6 it possible to see that if one disregards complex roots which determine the oscillatory character of the armature's motion ~~in the~~ relay, then a maximum reduced sensitivity of the apparatus of the order of 0.4 to 0.5 is obtained around a value of about 0.15 for the lumped constant, $CL^2/mR^2 \approx 0.15$. Then the explcitation frequency is determined by ~~from~~ the following ratio:

$$Lw_0/R \approx 0.55 \qquad (27^{III})$$

If one sets in the last expression $R = R_1 + R_2$, where R_1 is the resistance of the signal oscillator and R_2 is the resistance of the relay winding, it ~~then the following statements~~ will be almost exact when ~~hold true approximately~~:

$$W_0L = R_2 = R_1, \qquad (27^{IIII})$$

that is, if R_1 and R_2 are approximately the same. Below is given the theoretical foundation of these relations.

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Keeping in mind the conclusion reached in the author's first work involving the graphical method, one must do as follows, in order to design correctly a polarized relay:

1. make a definite design of the projected apparatus;

2. in accordance with the design selected, determine the reduced mass m of the armature;

3. select the ~~above-described~~ exploitation frequency W_o which can be ~~encountered~~ in practice for a given type of apparatus;

4. select the ~~capacity~~ of the signal ~~oscillator~~ ~~feeding~~ the apparatus and specify its internal resistance R; this ~~capacity~~ must be sufficient, but not excessive, to ~~overlap~~ the least ~~capacity~~ as determined by Equation (24);

5. select a ~~source of~~ permanent magnetism that ~~ensures~~ the relation $C/mw_o^2 \approx 0.52$;

6. select the number of turns of the signal winding ~~in accordance with~~ the condition $R/L\omega_o \approx 0.55$;

7. place the winding on the relay in such a way that the coefficient of scattering σ is a minimum, and magnetic 'concentration' is a maximum; then the resistance of the windings will be a minimum in comparison with the oscillator's resistance.

8. if any of these requirements contradict those preceding, than it is necessary to assign another dimension or another design for the apparatus.

11. Conclusions.

1. ~~the~~ A general linear theory of the operation of a two-position polarized apparatus was ~~established~~ constructed, based upon ~~by~~ a number of assumptions realizable in practice.

2. the instability of operation in those cases where the coefficient of stability and signal strength do not correspond to periodic solutions of the equations found was noted.

3. the critical case where the armature touches the stop in just a momentary collision was examined in detail.

4. a ~~scheme~~ of comparison chart was established, giving in the most general case the dependence of minimum power ~~capacity~~ necessary for a given ~~for the reduced~~ mass of the armature in oscillatory motion upon the frequency and amplitude of these oscillations.

5. various relations between the relay parameters determining the values of maximum sensitivity were calculated for the critical case.

6. dimensionless combinations, as functions of which the most important exploitation data was defined graphically, were formed from the relay parameters.

7. design procedure was followed for a two-position apparatus with maximum attainable sensitivity within the boundary of an oscillatory regime.

12. Supplement.

1. Correction factor: As seen from tables 1, 2, 3 and Figure 6 the most interesting region of the α, β-plane is the region around the point $\alpha = \beta = 2$. For example, the equivalent region around the point $\alpha = \beta = 1/2$ does not possess any practical value, since very great ~~self~~-inductances L correspond to this region for very small resistance R, which is already provided ~~situation the apparatus avoids~~ by proper the construction of the apparatus ~~designing~~.

Table 4 gives the numerical values of the ratio of operators in Formula (22) for the neighborhood of this point. A similar calculation carried out for an interesting region of complex roots gives analogous results. Approximate analytical evaluation of the operators for this region shows that the average magnitude of the correction due to this ratio in the region of maximum sensitivity of the apparatus is about $1/\sqrt{1+1} \approx 0.7$ The sensitivity shown in Tables 1, 2, 3 and Figure 6 must therefore be increased approximately by 40%.

2. Polarized two-position relay with spring: Let us assume that a weak spring is attached to the armature of the relay so that the spring tends to return the armature to its neutral position. Then the spring's positive elasticity (stiffness) C_1 will ~~be counter-acted by a~~ oppose the negative elasticity C of the magnetic field, so that only the resulting negative elasticity $C_2 = C - C_1$ will ~~play the~~ finally ~~role~~ be important.

This means that C_2 must replace C in Equation (3) and in all succeeding equations and that C_2 is less than C. Then one can obtain the fact that $(1-\sigma)^2$ will not be less but equal to or even greater than unity.

Figure 6 ~~depicts~~ shows the line corresponding to equal roots p_2 and p_1; $\alpha = \beta = a$; $b = o$ in the interval from $\sigma = +0.6$ to $\sigma = -1.15$. As we see, the use of a spring in this interval ~~increases~~ (approximately doubles) the sensitivity of the relay ~~about double~~ when the exploitation frequency has been reduced about one and a half times (see table 2).

\- E N D -

(All Figures and tables are annexed.)

Moscow State U.

Chair of Gen'l Physics

Submitted 10 Apr '47

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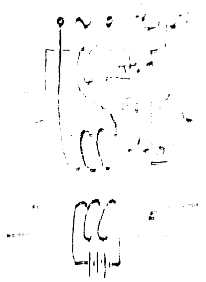

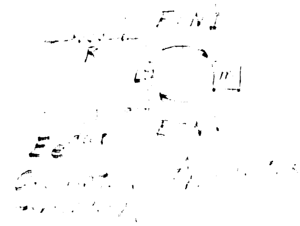

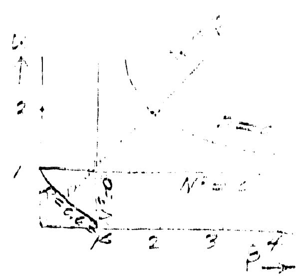

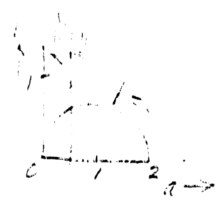

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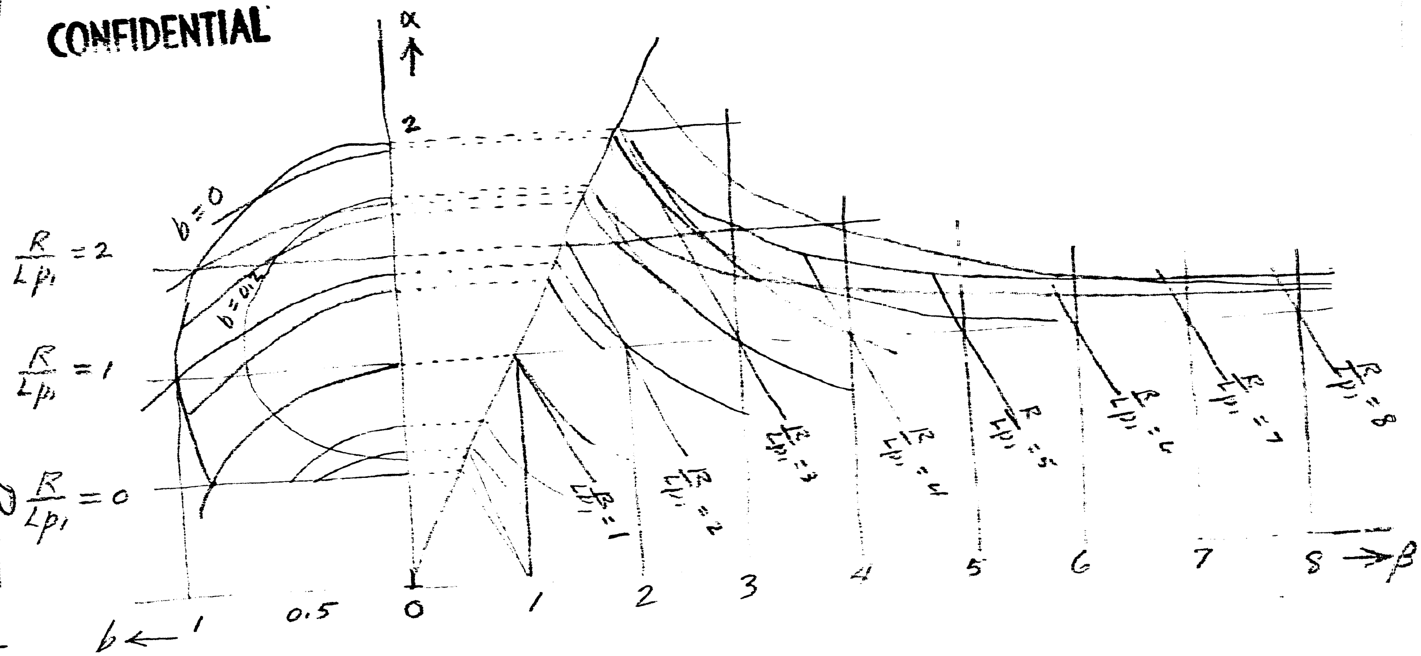

Figure 5 (page 373)

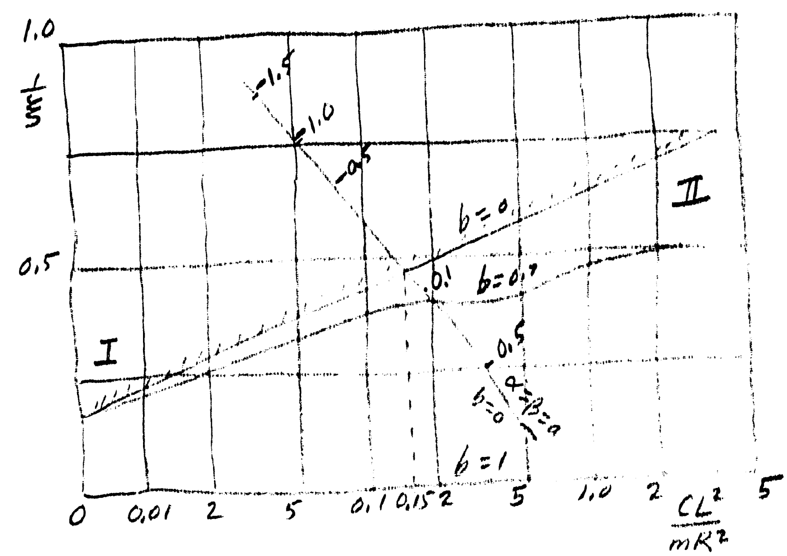

Figure 6 (p 374)

I : the region of real roots.
II : the region of complex roots.

Table 1 CONFIDENTIAL

Real Roots

0.5	0.5	0.4	0.63	0	1			—
2.0	2.0	2.57	1.60	0	0.461	0.535	0.520	0.148
1.77	2.3	2.57	1.60	0	0.459	0.521	0.516	0.140
1.667	2.5	2.60	1.61	0	0.442	0.508	0.505	0.132
1.5	3.0	2.68	1.64	0	0.435	0.469	0.480	0.105
1.333	4.0	2.80	1.67	0	0.392	0.386	0.439	0.0655
1.25	5.0	2.94	1.72	0	0.355	0.328	0.405	0.0435
1.139	6.3	3.00	1.73	0	0.320	0.268	0.387	0.0278
1.08	14.14	3.105	1.76	0	0.178	0.124	0.345	0.0053
1.75	1.75	2.14	1.46	0.2	0.382	0.585	0.575	0.197
1.56	2.0	2.15	1.47	0.2	0.380	0.575	0.568	0.187
1.41	2.5	2.24	1.50	0.2	0.398	0.516	0.541	0.144
1.29	3.0	2.38	1.54	0.2	0.382	0.470	0.495	0.109
1.20	4.0	2.58	1.61	0.2	0.340	0.383	0.448	0.065
1.04	14.0	3.0	1.73	0.2	0.171	0.123	0.346	0.00525

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Table 2

Equal Roots, b = 0

K=1 ea	Z	$\sqrt{Z}$		$\frac{1}{\lvert\xi\rvert}$	$\frac{\omega_0 L}{R}$	$\frac{C}{m\omega_0^2}$	$\frac{CL^2}{mR^2}$
0.5	0.4	0.63	0	1.000	∞	∞	—
0.8	0.74	0.86	0.688	0.156	1.43	1.44	2.94
1.35	1.5	1.22	0.57	0.213	0.720	0.715	0.370
1.67	1.67	2.0	0.27	0.364	0.603	0.593	0.220
1.87	2.35	1.53	0.10	0.424	0.559	0.545	0.170
1.96	2.5	1.58	0.029	0.451	0.540	0.525	0.153
2.0	2.57	1.60	0.0	0.461	0.535	0.520	0.148
2.24	3.0	1.73	-0.18[illegible]	0.528	0.498	0.480	0.119
2.72	4.0	2.0	-0.475	0.634	0,451	0.418	0.085
3.16	5.0	2.24	-0.72	0.704	-	-	0.0667
3.55	6.0	2.45	-0.91	0.730	-	-	0.0560
3.92	7.0	2.64	-0.08	0.788	-	-	0.0480
4.26	8.0	2.83	-1.23	0.813	-	-	0.0430
4.88	10.0	3.16	-1.48	0.862	-	-	0.0357

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Table 3

Complex Conjugate Roots

a	b	z	$\sqrt{z}$	σ	$\frac{1}{\lvert\varepsilon'\rvert m}$	$\frac{\omega_0 L}{R}$	$\frac{C}{m\omega_0^2}$	$\frac{CL^2}{mR^2}$
2.0	0.0	2.57	1.60	0	0.461	0.535	0.518	0.148
1.80	0.60	2.44	1.56	0	0.490	0.600	0.568	0.204
1.30	0.96	2.08	1.44	0	0.595	0.885	0.785	0.636
1.0	1.0	1.75	1.32	0	0.69	1.30	1.14	2.0
0.55	0.9	1.08	1.04	0	-	10.4	4.58	495.0
0.5	0.867	1.0	1.0	0	1.0	∞	∞	∞
0.6	0.8	1.0	1.0	0.02	0.813	5.0	5.00	125.0
0.8	0.6	1.0	1.0	0.2	0.500	1.67	1.67	4.65
1.0	0.686	1.33	1.15	0.2	0.481	1.15	1.28	1.69
1.2	0.68	1.60	1.26	0.2	0.453	0.90	1.07	0.87
1.6	0.425	2.0	1.41	0.2	0.388	0.64	0.88	0.36
1.75	0.0	2.14	1.46	0.2	0.373	0.585	0.84	0.284
1.64	0.3	2.0	1.41	0.233	0.373	0.621	0.86	0.332

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Table 4

Values of the Ratio T in Formula (22)

[illegible]	Γ	ω/ρ			[illegible]	Γ	ω/ρ		
		0.5	1.0	2.0			0.5	1.0	2.0
0.4	0.7	1.03	1.06	1.3	1.5	2.5	1.16	1.3	1.39
0.4	0.8	1.09	1.1	1.51	1.5	4.0	1.58	-	-
0.5	0.6	1.12	1.10	1.42	1.5	6.0	1.28	-	-
0.5	0.8	1.06	1.1	1.53	1.6	1.8	1.46	1.49	1.4
0.6	0.8	0.96	1.45	1.09	1.6	2.0	1.1	1.6	0.96
1.3	1.5	1.19	1.25	1.17	1.6	2.2	0.78	1.35	0.93
1.3	4.0	1.24	1.2	1.99	1.6	3.0	1.03	1.12	1.3
1.4	6.0	1.15	1.18	1.16	1.7	1.8	1.14	1.21	1.93
1.5	1.8	1.10	1.31	1.78	1.8	2.0	1.25	1.29	1.77
1.5	2.0	1.13	1.47	1.33	1.8	2.5	1.28	1.56	1.4

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